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ORIGINAL ARTICLE

Effect of rotation on plane waves at the free surface of a fibre-reinforced thermoelastic half-space using the finite element method

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Abstract The propagation of plane waves in fibre-reinforced, rotating thermoelastic half-space proposed by Lord-Shulman is discussed. The problem has been solved numerically using a finite element method. Numerical results for the temperature distribution, the displacement components and the thermal stress are given and illustrated graphically. Comparisons are made with the results predicted by the coupled theory and the theory of generalized thermoelasticity with one relaxation time in the presence and absence of rotation and reinforcement. It is found that the rotation has a significant effect and the reinforcement has great effect on the distribution of field quantities when the rotation is considered.

Keywords Rotation · Fibre-reinforced medium · Half-space · Thermal relaxation time · Finite element method

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1 Introduction

Fibre-reinforced composites are used in a variety of structures due to their low weight and high strength. The characteristic property of a reinforced composite is that its components act together as a single anisotropic unit as long as they remain in the elastic condition. Verma [1] discussed the problem of magnetoelastic shear waves in self-reinforced bodies. Chattopadhyay and Choudhury [2] investigated the propagation, reflection and transmission of magnetoelastic shear waves in a self-reinforced media. Chattopadhyay and Choudhury [3] studied the propagation of magnetoelastic shear waves in an infinite self-reinforced plate. Chattopadhyay and Michel [4] studied a model for spherical SH-wave propagation in self-reinforced linearly elastic media, In classical dynamical coupled theory of thermoelasticity, the thermal and mechanical waves propagate with an infinite velocity, which is not physically admissible. The theory of couple thermoelasticity was extended by Lord and Shulman (LS) [5] and Green and Lindsay [6] by including the thermal relaxation time in constitutive relations. During the last three decades a number of investigations [7–11] have been carried out using the aforesaid theories of generalized thermoelasticity. Note that in most of the earlier studies mechanical or thermal loading on the boundary surface was considered to be in the form of a shock.

The exact solution of the governing equations of the generalized thermoelasticity theory for a coupled and nonlinear/linear system exists only for very special and simple initial and boundary problems. To calculate the solution of general problems, a numerical solution technique is used. For this reason the finite element method is chosen. The method of weighted residuals offers the formulation of the finite element equations and yields the best approximate solutions to linear and nonlinear boundary and partial differential equations. Applying this method basically involves three steps. The first step is to assume the general behavior of the unknown field variables in such a way as to satisfy the given differential equations. Substitution of these approximating functions into the differential equations and boundary conditions results in some errors, called the residual. This residual has to vanish in an average sense over the solution domain. The second step is the time integration. The time derivatives of the unknown variables have to be determined by former results. The third step is to solve the equations resulting from the first and second step by using a finite element algorithm program (see Zienkiewicz [12]). Abbas [13], Abbas and Abd-Alla [14] and Youssef and Abbas [15] applied the finite element method in different problems.

In the present work, the (LS) theory is applied to study the influence of rein-forcement on the total deformation of rotating body and the interaction with each other. The problem has been solved numerically using a finite element method (FEM). Numerical results for the temperature distribution, displacement and the stress components are represented graphically.

2 Formulation of the problem and basic equations

The constitutive equations for a fibre-reinforced linearly thermoelastic anisotropic medium with respect to the reinforcement direction **a** are

$$\begin{aligned}\sigma_{ij} &= \lambda e_{kk}\delta_{ij} + 2\mu_T e_{ij} + \alpha(a_k a_m e_{km}\delta_{ij} + a_i a_j e_{kk}) \\ &\quad + 2(\mu_L - \mu_T)(a_i a_k e_{kj} + a_j a_k e_{ki}) \\ &\quad + \beta a_k a_m e_{km} a_i a_j - \gamma(T - T_0)\delta_{ij}. \end{aligned} \tag{1}$$

Where σ_{ij} are the components of stress; e_{ij} are the components of strain; λ, μ_T are elastic constants; $\alpha, \beta, (\mu_L - \mu_T)$ and γ are reinforcement parameters, T is the temperature above reference temperature T_0 and $\mathbf{a} \equiv (a_1, a_2, a_3)$, $a_1^2 + a_2^2 + a_3^2 = 1$. We choose the fibre-direction as $\mathbf{a} \equiv (1, 0, 0)$. The strains can be expressed in terms of the displacement u_i as

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}). \tag{2}$$

For plane strain deformation in the xy-plane, $\frac{\partial}{\partial z} \equiv 0$, $w = 0$.

Equation (1) then yields

$$\sigma_{xx} = A_{11}u_{,x} + A_{12}v_{,y} - \gamma(T - T_0), \tag{3}$$

$$\sigma_{yy} = A_{22}v_{,y} + A_{12}u_{,x} - \gamma(T - T_0), \tag{4}$$

$$\sigma_{zz} = A_{12}u_{,x} + \lambda v_{,y} - \gamma(T - T_0), \tag{5}$$

$$\sigma_{xy} = \mu_L(u_{,y} + v_{,x}), \qquad \sigma_{zx} = \sigma_{zy} = 0. \tag{6}$$

Where

$$\begin{aligned} A_{11} &= \lambda + 2(\alpha + \mu_T) + 4(\mu_L - \mu_T) + \beta, \\ A_{12} &= \alpha + \lambda, \qquad A_{22} = \lambda + 2\mu_T. \end{aligned} \tag{7}$$

Since the medium is rotating uniformly with an angular velocity $\mathbf{\Omega} = \Omega\mathbf{n}$, where $\mathbf{n}$ is a unit vector representing the direction of the axis of rotation. The displacement equation of motion in the rotating frame of reference has two additional terms (Schoenberg and Censor [17]): Centripetal acceleration, $\mathbf{\Omega} \times (\mathbf{\Omega} \times \mathbf{u})$ due to time-varying motion only and the Corioli's acceleration $2\mathbf{\Omega} \times \dot{\mathbf{u}}$ where $\mathbf{u}$ is the dynamic displacement vector. These terms don't appear in non-rotating media. The dynamic displacement vector is actually measured from a steady-state deformed position and the deformation is supposed to be small.

The equation of motion in a rotating frame of reference in the context of Lord-Shulman's theory is

$$\rho[\ddot{u}_i + \{\Omega \times (\Omega \times u)\}_i + (2\Omega \times \dot{u})_i] = \sigma_{ij,j} \quad (i, j = 1, 2, 3). \tag{8}$$

The heat conduction equation

$$kT_{,ii} = \left(\frac{\partial}{\partial t} + \tau_0 \frac{\partial^2}{\partial t^2}\right)(\rho C_E T + \gamma T_0 u_{i,i}). \tag{9}$$

Where ρ is the mass density, k is the thermal conductivity, C_E is the specific heat at constant strain and τ_0 is the relaxation time.

From (3)–(6) we note that the third equation of motion in (8) is identically satisfied and the first two equa-